Dynamic Analysis of a Kinematic Axis with Electromechanical Variator
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Abstract. The paper presents the mathematical model of a kinematic axis with a.c. motor drive and servodrive variator. The servodrive variator is composed of a differential mechanism and a friction variator. The influence of the load on the differential mechanism input elements will be analysed. In other papers, the authors analysed the behaviour of an ideal variator. In this paper was realised a mathematical model in which it was take in account the shift and the efficiency. Also, was realised an experimental model and a stand for comparing experimental and theoretical results. Using numerical methods, the differential equation will be solved and as a result, the response of the system for different input signals.

1. DESCRIPTION OF THE VARIATOR

The author have conceived and realised two types of electromechanical variators, which can be used to control the kinematic axis. The continuous regulation of the speed and the reversing of the revolution sense are assured through the variator command.

These solutions operate with asyncrone engines which function, at constant revolution, in all the stages of the dynamic regime. Thus, the inertia of the masses which are close to the engine is used during the transitory regime.

An electromechanical variator with external differential mechanism is shown in figure 1.

The cinematic function of the differential mechanism transfer has been used:

$$\omega_i = \omega_h \cdot (i + 1) - \omega_i \cdot i; \quad \omega_e = \omega_s \cdot i \cdot i = \frac{Z_1}{Z_3}. \quad (1)$$

The analysis of the different situations of functioning for the electromechanical variators leads to the conclusion that the resistant charge, generally noted $Q_{\text{e}}$, acts in different directions towards the entrance elements of the differential. The charge has either a motor or a resistant influence towards the two entrances (table 1.).

<table>
<thead>
<tr>
<th>Nr. crt.</th>
<th>$\omega_h$</th>
<th>$\omega_s$</th>
<th>$M_m$</th>
<th>$Q_{\text{e}}^s$</th>
<th>$M_1$</th>
<th>$M_S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>3</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>4</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
</tr>
</tbody>
</table>

Fig. 2. Energetical flow

This analyse is followed by the result that the flow of energy transmission contains a “closed loop of dead power”, which is produced by the charge itself (fig. 2).
The proposed structure can be used for one or more kinematic axis control (fig. 3 and 4).

Fig. 3. Kinematic axis for translation motion

Fig. 4. System of kinematic axis

2. THE MATHEMATICAL MODEL

For the differential mechanism \( \Sigma \), the theorem of the cynetic energy variation has been used:

\[
dL_{ext} = dE_c \\
M_\omega \omega \cdot dt - M_\omega \omega \cdot dt = Q^e_\omega \omega + d\left(\frac{1}{2} J_1 \omega^2\right).
\]

(3)

The displacement of the instant rotation centre in the plane movement of the \( z_2 \) satellite has been studied and the expressions of the moments on the differential elements have been obtained:

\[
M_i = \frac{1}{i_c} \left[ M_i + \eta_i \left(Q^e_i + J_i \omega_i\right) - J_i \omega_i \omega_i \right] - i_c J_i \omega_i.
\]

(4)

\[
M_e = \frac{1}{i_c} \left[ M_e + \eta_i \left(Q^e_i + J_i \omega_i\right) - J_i \omega_i \omega_i \right] - i_c J_i \omega_i.
\]

(8)

Based on this analysis and on relation (3), the differential mechanism has been decomposed and an equivalent scheme of the energetic flow has been built (fig. 3).

Fig. 5. Equivalent scheme

In order to introduce the variator shift with the friction and the efficiency of the transmission into the mathematical model the scheme from the figure 3 is used.

Denoting with \( \eta \), the variator efficiency and neglecting the inertial torque of the rolls and the ring [CIU 98b] could be written as the relation between the input and output variator torque:

\[
\eta_i \frac{M'_i}{C_{T_1}} \frac{1}{i_v} - C_{T_2} = M'_e.
\]

(5)

where:

\[
C_{T_1} = \begin{cases} 1 & \text{for } M'_i \leq R_c T_{max} \\ \frac{M_0}{M'_i} & \text{for } M'_i > R_c T_{max} \end{cases}
\]

(6)

\[
C_{T_2} = \begin{cases} 1 & \text{for } C_{T_1} i_v \leq R_c T_{max} \\ \frac{M_0}{C_{T_1} M'_i} i_v & \text{for } C_{T_1} i_v > R_c T_{max} \end{cases}
\]

(7)

The functions \( C_{T_1} \) and \( C_{T_2} \) bound the force values and the torque transmitted by the variator.

Taking into account the transmission efficiencies of the motion implied by the differential mechanism and using the kinematical energy variation theorem the input and output torques for friction variator could be obtained:

\[
M'_i = \frac{1}{i_c} \left[ M_i + \eta_i \left(Q^e_i + J_i \omega_i\right) - J_i \omega_i \omega_i \right] - i_c J_i \omega_i
\]

\[
M'_e = \frac{1}{i_c} \left[ M_e + \eta_i \left(Q^e_i + J_i \omega_i\right) - J_i \omega_i \omega_i \right] - i_c J_i \omega_i.
\]
By using the kinematical transfer function (1) we get the relation:

\[
M^*_i = \frac{i_{c2}(i+1)}{\eta_s} \left( Q_m^* + J_i \dot{\omega}_i \right) + \frac{1}{i_{c2}(i+1)} J_s \left( \dot{\omega}_i + i \dot{\omega}_e \right) + \frac{1}{i+1} J_s \left( \dot{\omega}_i + i \dot{\omega}_e \right)
\]

With the relations (5), (8) and (9) the motion differential equation could be obtained:

\[
C_{\lambda1} C_{\lambda2} \eta_s \left\{ \frac{1}{i_{c2}} \left[ M_i + \eta_i \left( Q_m^* + J_i \dot{\omega}_i \right) - J_i \dot{\omega}_i \right] - \frac{i_{c2}(i+1)}{\eta_s} \left( Q_m^* + J_i \dot{\omega}_i \right) + \frac{1}{i_{c2}(i+1)} \cdot J_s \left( \dot{\omega}_i + i \dot{\omega}_e \right) + \frac{1}{i+1} J_s \left( \dot{\omega}_i + i \dot{\omega}_e \right) \right\}
\]

The previous differential equation contains as unknown variables the angular velocities \( \omega_i(t) \) and \( \omega_e(t) \). The dependency between \( \omega_i(t) \) and \( \omega_e(t) \) given by the relation (1) couldn’t be used further because of the variator shift, but could be used for getting the derivate of the \( \omega_i(t) \) taking into account the following hypothesis:

**During the variator shift the angular velocity \( \omega_i \) doesn’t modify because of the limitation of the variator torque.**

Based on the hypothesis and on the relation (1) we can write the relation:

\[
\dot{\omega}_i = \frac{\dot{\omega}_e - C_{\omega^*} i_v^*}{(c_i - i)^3} C_{\omega^*}, \quad (11)
\]

where:

\[
C_{\omega^*} = \begin{cases} 
1, & \text{for } C_{\lambda1} C_{\lambda2} = 1 \\
0, & \text{for } C_{\lambda1} C_{\lambda2} \neq 1
\end{cases} \quad (12)
\]

Knowing the input angular velocity at the initial time \( t=0 \) the following relation could be obtained:

\[
\omega_i(t) = \frac{1}{2} \left[ \omega_i(t=0) + \int_0^t \dot{\omega}_i(t) \, dt + \frac{\dot{\omega}_e - C_{\omega^*} i_v^*}{(c_i - i)^3} \right]. \quad (13)
\]

The relations (10), (11) and (13), together with the conditions (6), (7) and (12) form a system with differential equations which describes the mechanical system behavior.

### 3. THE RESPONSE OF THE SYSTEM TAKING INTO ACCOUNT THE VARIATOR SHIFT

The software tool “Mathcad 2000” was used to model the mechanical system behavior.

#### 3.1. The variator behavior for a ramp input signal

The response time and the system stability were studied using as control function the transfer rate of the variator with friction:

\[
i_v(t) = \begin{cases} 
i_v(0) + m \cdot t & \text{for } t \leq 0 \\
i_{v_{\text{max}}} & \text{for } t > 0
\end{cases} \quad (14)
\]

The graphics from the table 2 were obtained by assigning the specified values.

**Table 2. Ramp input signal**

<table>
<thead>
<tr>
<th>( M_{\text{max}} ) [Nm]</th>
<th>( M_e = 0 ) [Nm]</th>
<th>( m = 2 ) [t]</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>0</td>
<td>200</td>
</tr>
<tr>
<td>50</td>
<td>0</td>
<td>200</td>
</tr>
</tbody>
</table>

Using the control function:

\[
i_v(t) = \begin{cases} 
1 & \text{for } t = 0 \\
1.2 & \text{for } t \geq 0
\end{cases} \quad (15)
\]

the results presented in the table 3 were obtained.

**Table 3. Ramp input signal**

<table>
<thead>
<tr>
<th>( M_{\text{max}} ) [Nm]</th>
<th>( M_e = 0 ) [Nm]</th>
<th>( m = 2 ) [t]</th>
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<tbody>
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</tr>
<tr>
<td>50</td>
<td>0</td>
<td>200</td>
</tr>
</tbody>
</table>

#### 3.2. The variator behavior for a unit step input signal

Using the control function:

\[
i_v(t) = \begin{cases} 
1 & \text{for } t = 0 \\
1.2 & \text{for } t \geq 0
\end{cases} \quad (15)
\]
3.3. The variator behavior for a sinusoidal input signal

With the control function:

$$i_s(t) = i(0) + A \sin(a t + b), \quad (16)$$

the model of the system gives the results presented in the table 4.

Table 4. Sinusoidal input signal

<table>
<thead>
<tr>
<th>Mmax=25[Nm]</th>
<th>Me=0</th>
<th>t[ms]</th>
</tr>
</thead>
<tbody>
<tr>
<td>i(0)=1.2</td>
<td>A=0.1</td>
<td>a=20</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mmax=25[Nm]</th>
<th>Me=0</th>
<th>t[ms]</th>
</tr>
</thead>
<tbody>
<tr>
<td>i(0)=1.2</td>
<td>A=0.1</td>
<td>a=50</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mmax=50[Nm]</th>
<th>Me=0</th>
<th>t[ms]</th>
</tr>
</thead>
<tbody>
<tr>
<td>i(0)=1.2</td>
<td>A=0.1</td>
<td>a=50</td>
</tr>
</tbody>
</table>

4. EXPERIMENTAL RESULTS

For the validation of results, the authors are realized an experimental stand based of the basic block diagram (fig. 5.).

With the Lab View program (fig. 6 and 7) the system gives the results as those shown in the figure 8.
Fig. 6 Program for output signal

Fig. 4 The response of the system for ramp input signal (experimental)

4. CONCLUSIONS

Analysing the graphics presented in the tables 2, 3 and 4, the following have been established:

- the system is stable and the output signal is delayed related to the input signal for all the studied cases;
- the duration of the transitory regime is increased with the decrease of the building-up time of the input signal;
- the delay time has a longer duration (100-500 milliseconds) than the previous model, which is explained by the fact that the system couldn’t use all the inertial masses (because of the shift of the variator);
- the building-up time has the same order as the delay time which means that during this period the system gets energy from the motor as well as from the inertial masses;
- the stationary error is in the acceptable limits (below 5%), which is explained by the fact that increasing the motor shift is related to increasing of the dynamical charge;
- for the sinusoidal input signal, the output signal isn’t delayed from the previous one, but a crooked signal is obtained; the dynamics is improved with the increasing of the difference between the nominal torque and the charge related to the decreasing of the amplitude and frequency of the input signal;
- the charge value influences the duration of the transitory regime;
- a resistant charge stable in time with the value close to motor torque increases the stationary error;
- a resistant charge which is maintained in time close to the motor torque increases the stationary error;
- a resistant charge which is maintained in time close to the maximum couple of the variator leads to the impossibility of the control of the system;

REFERENCES
